Bodies in Equilibrium

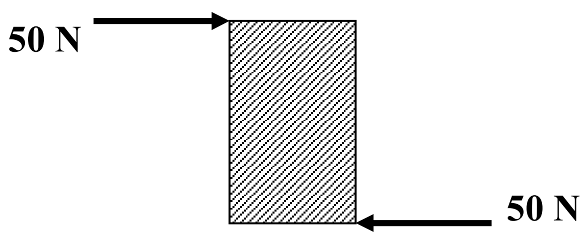
Recall Newton's First Law: if there is *no* unbalanced force on a body (i.e. if $\mathbf{F}_{\mathbf{Net}} = \mathbf{0}$), the body is in equilibrium. That is, if a body is in equilibrium, then all the forces on it must add up to equal zero; there is no resultant net force vector.

Based on this law, a body can be in equilibrium (i.e., $\mathbf{F}_{\mathbf{Net}} = \mathbf{0}$) under two conditions:

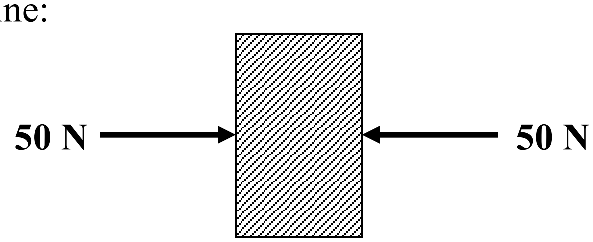
- It is stationary – this is known as *static equilibrium*.
- It is moving at a constant speed in a straight line – the object is in *dynamic equilibrium*.

However, even if these two conditions exist, an object still may not be truly in equilibrium. For example, the term "stationary" refers to an object that remains in place, not travelling from one position to another – the net force is zero. But if those forces act in different places on the object, it could start to rotate in place.

Consider two students pushing with equal force on opposite sides of a desk. Looking top-down on the desk:



You can see that although the forces cancel out, the desk will still move, by rotating. In order for these opposing forces to truly prevent motion, they must act along the same line:



To be more specific, if the line of action of each force acting on the body passes through a common point, the forces are termed *concurrent*. This is true regardless of how many forces act on the body.

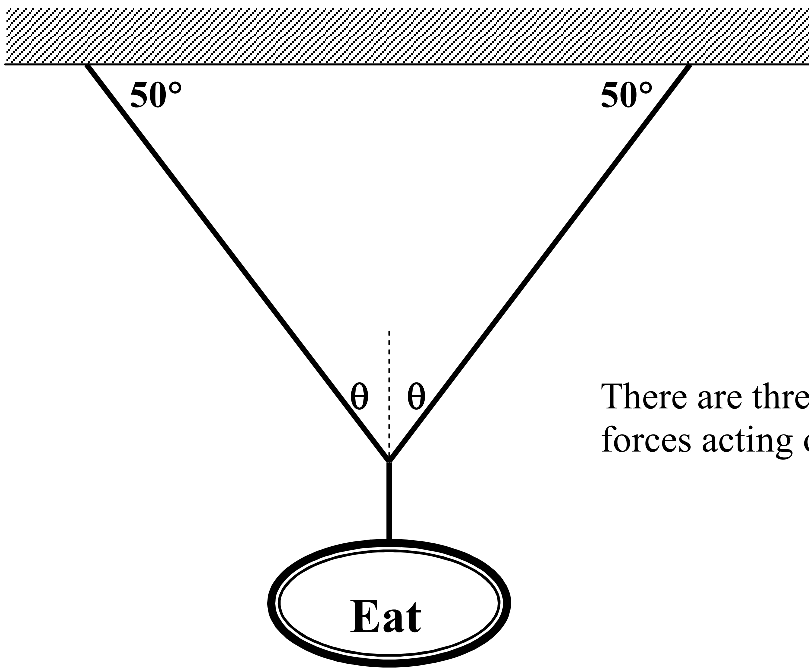
It can therefore be stated that if the net force on an object is zero, and if all the forces that act on the object are concurrent, then the object is in both *static* and *rotational equilibrium*.

For now, we will only look at simple three-force problems. When three concurrent forces act on an object that is in static equilibrium, the vector addition (tip-to-tail) diagram of these 3 vectors forms a **triangle**. Problems such as these should be solved in the following manner:

- <u>Identify</u> the common point where forces act concurrently.
- Draw a free-body diagram of the force vectors to show their orientation. This should look somewhat like a "Y" shape.
- Use the "tip-to-tail" method to draw a vector triangle; the tricky part here will be to label angles correctly.

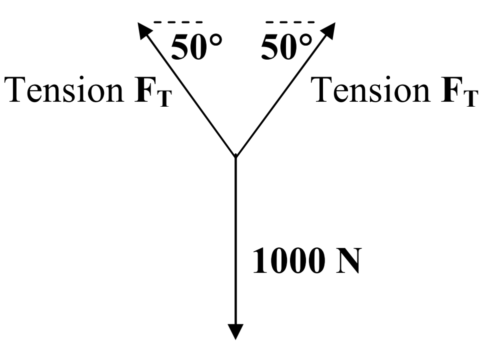
Consider the following problem: a sign is supported by two cables at equal angles as shown in the diagram. Find the tension in each cable. The sign weighs 1.0×10^3 N, and the cables make an angle of 50º with the support.

Note: tension is a <u>pulling</u> force only. Cables or ropes can *only* provide tension.

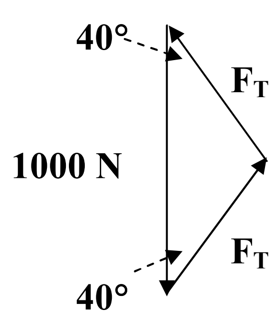


There are three concurrent forces acting on point '**P**'.

free-body diagram of forces:



Resultant vector triangle:



In this case both sides F_T are equal because we have an isosceles triangle. As well, by examining the vector triangle, we are given one side and the two included angles and asked to find one of the other sides. Therefore, use the Sine Law to solve (check formula sheet for reference):

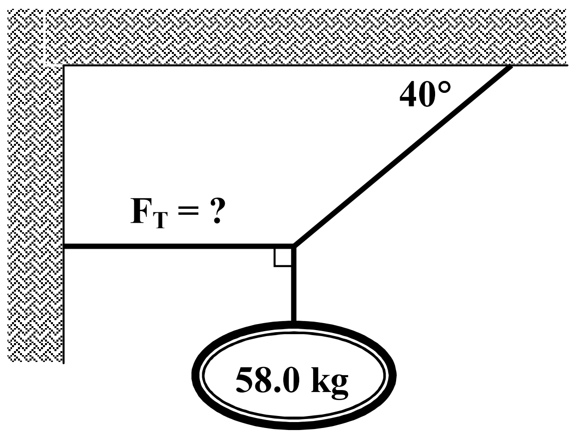
$$\frac{a}{\sin A} = \frac{b}{\sin B} \qquad \frac{T}{\sin 40^o} = \frac{1000}{\sin 100^o} \qquad \rightarrow \quad T = 6.5 \times 10^2 \text{ N}$$

An alternate solution is to solve using components in the 'x' and 'y' directions. Note the following:

- $\Sigma x_{left} = \Sigma x_{right}$
- $\Sigma y_{up} = \Sigma y_{down}$

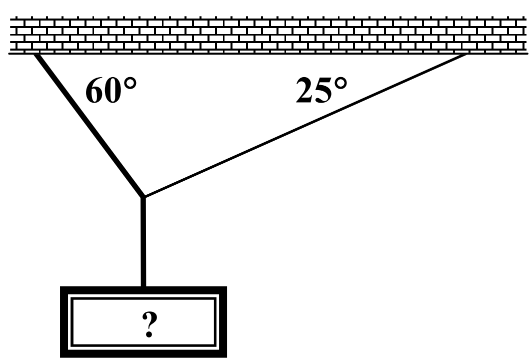
In each of the following examples, start by labelling the point where forces act concurrently, then draw the f.b.d., then the triangle with correct angles before solving.

Example # 1. Find the tension F_T in the horizontal cable.



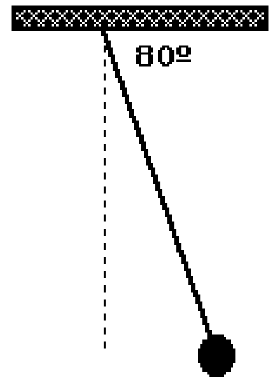
(see Equilibrium Ex 1 for answer)

Example #2. In this case, the left cable is strong, but the right cable can only handle a maximum tension of 5.0×10^2 N. What is the heaviest weight that can be hung?



(see Equilibrium Ex 2 for answer)

Example #3. A large mass of 50 kg is supported on the end of a rope and the rope is pulled back by a horizontal force so that the rope makes an angle of 80º with the ceiling to which the rope is attached. Make a forces diagram showing all the forces involved. Use this diagram to calculate what horizontal force is needed to pull the mass out this far.

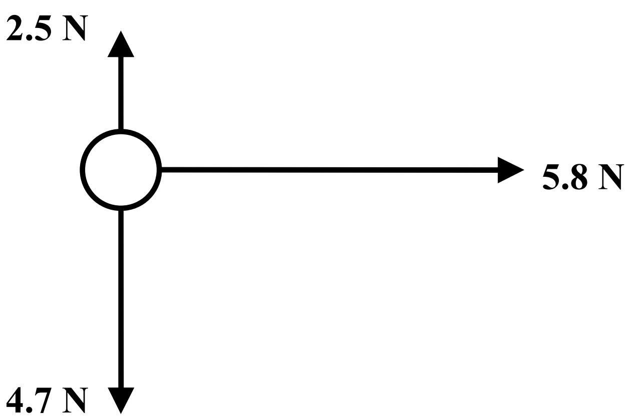


(see Equilibrium Ex 3 for answer)

Note that the force you are trying to find in order to hold an object in place is called the *equilibriant*. Sometimes, the forces in a situation do not add up to zero, and therefore are not in equilibrium, but would be if we added just one more force. To find that force in a concurrent forces situation (a) add up all the forces acting to find the resultant net force. (b) The equilibriant must be equal to this resultant, but in the opposite direction.

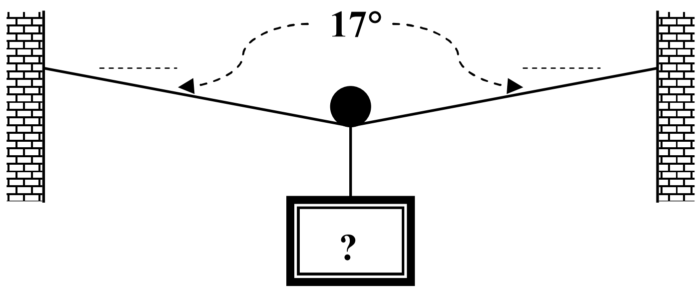
Example #4. These three forces act on a ball, as shown. Find:

a) **the unbalanced force on the ball;**
b) **the force that needs to be added to cause the ball to be in equilibrium.**



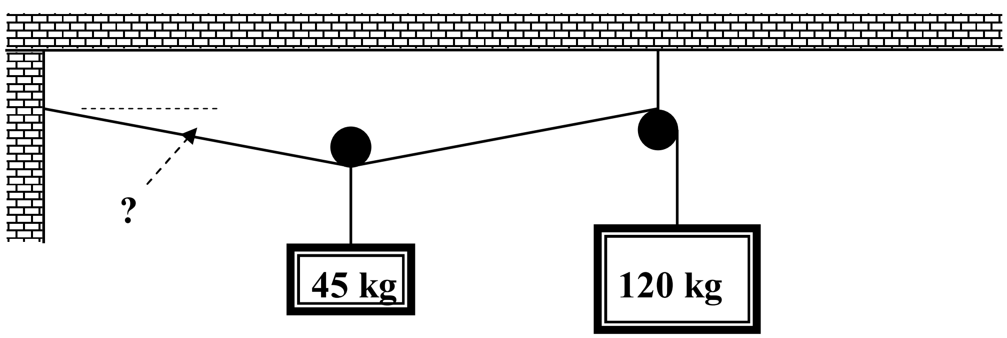
(see Equilibrium Ex 4 for answer)

Example #5. A cable that can wthstand a maximum tension of 850 N is strung across two walls. A pulley is placed on the cable, and various masses are hung from the pulley, causing the cable to sag. Through trial-and-error, it is found that the cable can only sag 17° without breaking. What mass was used to cause this sag? Note that a pulley can only change the direction of a cable; the tension on either side remains the same.



(see Equilibrium Ex 5 for answer)

Example #6. Now we'll take the previous example and vary it slightly. In the case below, the system is in equilibrium. Determine the unknown angle as indicated.



(see Equilibrium Ex 6 for answer)